

## Math 116 Section 04

Midterm 2

Name \_\_\_\_\_

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Student Number \_\_\_\_\_

All solutions are to be presented on the paper in the space provided. The exam is closed book, no calculators. Time for the exam is 50 minutes.

- (1) (**5 marks**) Find the area between the curves  $y = (x-1)^2$ ,  $y = 0$  and  $x = 0$ .

$$\begin{aligned} A &= \int_0^1 (x-1)^2 dx \\ &= \frac{1}{3}(x-1)^3 \Big|_0^1 \\ &= \frac{1}{3}((1-1)^3 - (0-1)) \\ &= \frac{1}{3} \end{aligned}$$

- (2) (**5 marks**) Find the volume obtained by rotating the region bounded by  $y = \sin x, y = 0, x = 0, x = \pi$  about the  $x$ -axis.

By disks:

$$\begin{aligned} V &= A(x) dx \\ &= \int_0^\pi \pi r^2 dx \\ &= \pi \int_0^\pi \sin^2 x dx \\ &= \pi \int_0^\pi \frac{1}{2}(1 - \cos(2x)) dx \\ &= \frac{\pi}{2} \left( x - \frac{1}{2} \sin(2x) \right) \Big|_0^\pi \\ &= \frac{\pi}{2} \left( \pi - \frac{1}{2} \sin(2\pi) - \left( 0 - \frac{1}{2} \sin 0 \right) \right) \\ &= \frac{\pi^2}{2} \end{aligned}$$

- (3) (**5 marks**) A spring has natural length 0.02m and spring constant  $10 \text{ kg s}^{-2}$ . How much work is required to stretch the spring to a length of 0.03m?

$$\begin{aligned} W &= \int_a^b F(x) dx \\ &= \int_{0.02}^{0.03} k(x - x_0) dx \\ &= \int_{0.02}^{0.03} 10(x - 0.02) dx \\ &= 10 \left. \frac{(x - 0.02)^2}{2} \right|_{0.02}^{0.03} \\ &= 10 \left( \frac{(0.03 - 0.02)^2}{2} - \frac{(0.02 - 0.02)^2}{2} \right) \\ &= 0.0005 \text{N} \end{aligned}$$

- (4) (**5 marks**) Find the average value of  $f(x) = \frac{1}{x}$  over the interval  $[\frac{1}{2}, 2]$

$$\begin{aligned} f_{\text{ave}} &= \frac{1}{b-a} \int_a^b f(x) \, dx \\ &= \frac{1}{2 - \frac{1}{2}} \int_{\frac{1}{2}}^2 \frac{1}{x} \, dx \\ &= \frac{2}{3} \ln x \Big|_{\frac{1}{2}}^2 \\ &= \frac{2}{3} \left( \ln 2 - \ln \frac{1}{2} \right) \\ &= \frac{2}{3} (\ln 2 + \ln 2) \\ &= \frac{4}{3} \ln 2 \end{aligned}$$